

# Hierarchy Problem, Dilatonic Fifth Force, and Origin of Mass

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Based on the fact that the scalar curvature of the internal space determines the mass of the dilaton in higher-dimensional unified theories, we show how the dilaton mass can explain the origin of mass and resolve the hierarchy problem. Moreover, we show that cosmology puts a strong constraint on dilaton mass, and requires the scale of the internal space to be larger than  $10^{-9} m$ .

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One of the fundamental problems in physics is the so called hierarchy problem. It has been very difficult to understand why the Planck mass fixed by the Newton's constant is so large compared to the mass scale of ordinary elementary particles, or equivalently why the gravitational force is so weak compared to other forces. There have been many proposals to resolve this problem. Long time ago Dirac conjectured that the Newton's constant may not be a constant but actually a time-dependent parameter to resolve the problem [1, 2]. This is an attractive proposal, because this conjecture can naturally be implemented in realistic higher-dimensional unified theories [3, 4]. Another interesting proposal based on the higher-dimensional unification is that the gravitational force in higher-dimension is actually as strong as other forces, but a relatively large (compared to the Planck scale) internal space of the order of  $TeV$  scale makes the 4-dimensional gravitational force very weak [5, 6].

A totally independent on the surface but actually intimately related problem is the dilatonic fifth force and the origin of dilaton mass [7, 8]. All existing higher-dimensional unified theories (Kaluza-Klein theory, supergravity, and superstring) predict the existence of the dilaton, the fundamental scalar graviton which couples to all matter fields [3, 9, 10]. As the scalar graviton it generates a fifth force which can affect the Einstein's gravity in a fundamental way, and has a deep impact in cosmology. The reason why the dilatonic fifth force is related to the hierarchy problem is that the dilaton mass which deter-

mines the range of the fifth force is given by the scalar curvature of the internal space, whose scale is fixed by the scale of the internal space [4, 7]. *The purpose of this Letter is to show how the dilaton mass can resolve the hierarchy problem and allow us to understand the origin of mass, and to discuss how the cosmology excludes the dilaton with mass heavier than 160 eV.* This, together with the known fifth force experiment constraint on dilaton mass, requires the scale of internal space to be smaller than  $10^{-5} m$ , but larger than  $10^{-9} m$ .

It has been well known that in higher-dimensional unified theories the volume element of the internal space has important roles in 4-dimensional physics [3, 4]. It becomes a fundamental scalar graviton known as the Kaluza-Klein dilaton, which makes the Newton's constant space-time dependent and naturally realizes the Dirac's conjecture [4, 7]. As an essential part of the  $(4 + n)$ -dimensional metric the dilaton (also called the "radion" [6] or the "chameleon" [11]) couples to all matter fields, and generates the dilatonic fifth force which could compromise the equivalence principle [8, 12]. It acquires a mass geometrically, and thus can easily accommodate the experimental fact that there is no long range fifth force in nature [13, 14]. Moreover, in cosmology the dilaton could play the role of the inflaton, and well be the dark matter of the universe [12, 15]. But as the dark matter it can not have arbitrary mass. This severely constrains the dilaton mass, and thus the size of the internal space. In the following we discuss how the dilaton acquires a mass and how the existing constraints on the dilaton mass restrict the size of the internal space.

To do this, it is worth comparing two dimensional reduction methods in higher-dimensional theories, because the way the dilaton couples to matter fields crucially depends on the dimensional reduction methods. In the

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popular dimensional reduction one treats the  $(4+n)$ -dimensional space as physical and view the 4-dimensional physics as an approximation in the limit when the  $n$ -dimensional space is very small [9, 10]. On the other hand, in the dimensional reduction by isometry, one assumes an exact isometry in the  $(4+n)$ -dimensional space to make the  $n$ -dimensional space unphysical [3, 4]. In this view the  $n$ -dimensional space becomes invisible not because of the small size but because of the isometry, so that the size of the  $n$ -dimensional space can be arbitrary in principle. It has been thought that the popular view is more general because it practically reduces to the other view in the zero mode approximation (where one excludes the massive Kaluza-Klein modes). However, there are subtle but important differences between the two reduction methods which can change the 4-dimensional physics drastically, as we will see in the following.

Since all higher-dimensional unified theories contain the  $(4+n)$ -dimensional gravity we start from the dimensional reduction of Kaluza-Klein theory. Consider the popular dimensional reduction first. In the zero mode approximation, one has to exclude the massive Kaluza-Klein modes. A simple way to do this is to introduce an  $n$ -dimensional isometry  $G$ , and to view the  $(4+n)$ -dimensional unified space as a principal fiber bundle  $P(M, G)$  made of the 4-dimensional space-time  $M$  as the base manifold and the  $n$ -dimensional group manifold  $G$  as the vertical fiber (the internal space) on which  $G$  acts as an isometry. Let  $\gamma_{\mu\nu}$  and  $\phi_{ab}$  be the metric on  $M$  and  $G$ ,  $\gamma$  and  $\phi$  be the determinants of  $\gamma_{\mu\nu}$  and  $\phi_{ab}$ , and  $\rho_{ab} = \phi^{-1/n} \phi_{ab}$  ( $|\det \rho_{ab}| = 1$ ) be the normalized internal metric. In this setting the  $(4+n)$ -dimensional Einstein-Hilbert action on  $P$  leads to the following 4-dimensional Lagrangian in the Jordan frame [2, 3]

$$\begin{aligned} \mathcal{L}_0 = & -\frac{\hat{V}_G}{16\pi G_P} \sqrt{\gamma} \sqrt{\phi} \left[ R_M \right. \\ & -\frac{n-1}{4n} \gamma^{\mu\nu} \frac{(\partial_\mu \phi)(\partial_\nu \phi)}{\phi} + \frac{\kappa^2}{4} \sqrt[n]{\phi} \rho_{ab} \gamma^{\mu\alpha} \gamma^{\nu\beta} F_{\mu\nu}^a F_{\alpha\beta}^b \\ & + \frac{\gamma^{\mu\nu}}{4} (D_\mu \rho^{ab})(D_\nu \rho_{ab}) + \frac{1}{\kappa^2 \sqrt[n]{\phi}} \hat{R}_G(\rho_{ab}) \\ & \left. + \Lambda_P + \lambda(|\det \rho_{ab}| - 1) \right], \end{aligned} \quad (1)$$

where  $G_P$  is the  $(4+n)$ -dimensional gravitational constant,  $\hat{V}_G$  is the normalized volume of the internal space  $G$ ,  $R_M$  is the scalar curvature of  $M$  fixed by  $\gamma_{\mu\nu}$ ,  $\hat{R}_G(\rho_{ab})$  is the normalized internal curvature of  $G$  fixed by  $\rho_{ab}$ ,

$$\hat{R}_G(\rho_{ab}) = -\frac{1}{2} f_{ab}^d f_{cd}^b \rho^{ac} - \frac{1}{4} f_{ab}^m f_{cd}^n \rho^{ac} \rho^{bd} \rho_{mn},$$

$\kappa$  is the unit scale of the internal space,  $F_{\mu\nu}^a$  is the gauge field of the isometry group,  $\Lambda_P$  is the  $(4+n)$ -dimensional cosmological constant, and  $\lambda$  is a Lagrange multiplier. To clarify the meaning of (1), we introduce the Pauli metric (also called the Einstein metric in string theory)  $g_{\mu\nu}$  and

the Kaluza-Klein dilaton  $\sigma$  by

$$\begin{aligned} g_{\mu\nu} &= \exp\left(\sqrt{\frac{n}{n+2}} \sigma\right) \gamma_{\mu\nu}, \\ \phi &= \left[v \exp\left(\sqrt{\frac{n}{n+2}} \sigma\right)\right]^2, \quad \langle \phi \rangle = v^2. \end{aligned} \quad (2)$$

With this (1) is expressed in the Pauli frame as [3, 12]

$$\begin{aligned} \mathcal{L} = & -\frac{v \hat{V}_G}{16\pi G_P} \sqrt{g} \left[ R + \frac{1}{2} (\partial_\mu \sigma)^2 - \frac{1}{4} (D_\mu \rho^{ab})(D^\mu \rho_{ab}) \right. \\ & + \frac{1}{\kappa^2} v^{-2/n} \hat{R}_G(\rho_{ab}) \exp\left(-\sqrt{\frac{n+2}{n}} \sigma\right) \\ & + \Lambda_P \exp\left(-\sqrt{\frac{n}{n+2}} \sigma\right) \\ & + \lambda \exp\left(-\sqrt{\frac{n}{n+2}} \sigma\right) (|\det \rho_{ab}| - 1) \\ & \left. + \frac{\kappa^2}{4} v^{2/n} \exp\left(\sqrt{\frac{n+2}{n}} \sigma\right) \rho_{ab} F_{\mu\nu}^a F^{b\mu\nu} \right]. \end{aligned} \quad (3)$$

This describes the well-known unification of gravitation with the gauge field, if one identifies [3, 4]

$$\frac{v \hat{V}_G}{16\pi G_P} = \frac{1}{16\pi G}, \quad \frac{\kappa^2}{16\pi G} = 1, \quad (4)$$

where  $G$  is the Newton's constant.

This tells the followings. First, although the unit scale of the internal space  $\kappa$  is fixed by the Planck scale  $\sqrt{16\pi G}$ , the vacuum expectation value of the volume of the internal space is given by [7, 8]

$$\langle V_G \rangle = v \hat{V}_G \simeq v \kappa^n = v (16\pi G)^{n/2}. \quad (5)$$

This means that the actual scale of the internal space  $L_G$  is given by  $v^{1/n} \kappa$ , not the Planck scale. Perhaps more importantly, this tells that the scale of the higher-dimensional gravitational constant  $G_P$  is given by

$$G_P^{1/(n+2)} = (16\pi)^{n/2(n+2)} v^{1/(n+2)} G^{1/2}. \quad (6)$$

This is precisely the equation which has been proposed to resolve the hierarchy problem [5, 6], which shows that a large  $v$  can easily bring  $G_P$  to the order of the elementary particle scale. Of course, in the popular dimensional reduction in which the  $(4+n)$ -dimensional space is treated as physical, the internal space can not assume a large scale because it has to be invisible at present energy scale. For this reason the size has often been assumed to be of the Planck scale, with  $v = 1$  [3, 9, 10]. But we emphasize that a relatively large internal space has not been ruled out theoretically as well as experimentally [7, 13].

Second, it is the Pauli metric  $g_{\mu\nu}$ , not the Jordan metric  $\gamma_{\mu\nu}$ , which describes the massless spin-two graviton of Einstein's theory [12]. But notice that (in the absence of the dilaton) its coupling to the gauge field depends on  $v$ , so that the graviton couples to the gauge

field non-minimally when  $v \neq 1$ . And this leads to a violation of the equivalence principle [8, 12]. Fortunately we can remove this  $v$ -dependent gravitational coupling by renormalizing  $F_{\mu\nu}^a$  to  $\hat{F}_{\mu\nu}^a = v^{1/n} F_{\mu\nu}^a$  and identifying  $\hat{F}_{\mu\nu}^a$  as the 4-dimensional gauge field. When one has higher-dimensional matter fields (as is the case in superstring or supergravity), however, the rescaling of the matter fields to absorb the  $v$ -dependent gravitational coupling is no longer possible in the popular dimensional reduction. This is because the normalization of the matter fields is pre-fixed by the  $(4+n)$ -dimensional physics, since the higher-dimensional space is treated as physical here [5, 6]. In this case a large  $v$  inevitably leads to a disastrous violation of the equivalence principle in Einstein's theory, which implies that the scale of the internal space can not be much larger than the Planck scale. This is a potentially dangerous defect of the popular dimensional reduction which has to be treated very carefully.

However, one can avoid this violation of the equivalence principle if one adopt the dimensional reduction by isometry. This is because in this case one can renormalize the matter fields and their masses with impunity after the dimensional reduction to absorb the  $v$ -dependent gravitational coupling, since only the 4-dimensional space-time is treated as physical in this dimensional reduction. In this case one can have a large  $v$  without violating the equivalence principle. This tells that the dimensional reduction by isometry has a logical advantage over the popular dimensional reduction [3, 16].

Suppose the Lagrangian (3) has the unique vacuum at  $\langle g_{\mu\nu} \rangle = \eta_{\mu\nu}$ ,  $\langle \sigma \rangle = 0$ ,  $\langle \rho_{ab} \rangle = \delta_{ab}$ ,  $\langle A_\mu^a \rangle = 0$ . With this we have the following dilaton potential  $V(\sigma)$ ,

$$V(\sigma) = v^{-2/n} \frac{\hat{R}_G}{(16\pi G)^2} \left[ \exp\left(-\sqrt{\frac{n+2}{n}} \sigma\right) - \frac{n+2}{n} \exp\left(-\sqrt{\frac{n}{n+2}} \sigma\right) + \frac{2}{n} \right], \quad (7)$$

where  $\hat{R}_G = \hat{R}_G(\delta_{ab})$  is the (dimensionless) curvature of  $G$  created by the Cartan-Killing metric  $\delta_{ab}$ . Notice that the potential is completely fixed by the vacuum condition  $V(0) = 0$  and  $dV(0)/d\sigma = 0$ . Clearly  $V(0) = 0$  assures that we have no *ad hoc* 4-dimensional cosmological constant, and  $dV(0)/d\sigma = 0$  shows that the  $(4+n)$ -dimensional cosmological constant  $\Lambda_P$  is uniquely fixed. From (7) we find the mass of the Kaluza-Klein dilaton  $\mu$ ,

$$\mu^2 = -v^{-2/n} \frac{\hat{R}_G}{8\pi n} m_p^2 = \frac{2}{n+2} \Lambda_P, \quad (8)$$

where  $m_p$  is the Planck mass. This demonstrates that a large  $v$  naturally transforms the large Planck mass to a small dilaton mass, which can easily be of the order of the elementary particle mass scale. This is the resolution of the hierarchy problem in Kaluza-Klein unification [4, 7]. But notice that  $\mu = 0$  when  $\hat{R}_G = 0$ , independent of  $n$  and  $v$ . From (8) we have the following scale of the

internal space  $L_G$  (when  $\hat{R}_G \neq 0$ ),

$$L_G = v^{1/n} \kappa = \sqrt{-\frac{2\hat{R}_G}{n}} \frac{1}{\mu} \simeq \frac{1}{\mu}. \quad (9)$$

So, for the  $S^3$ -compactification of 3-dimensional internal space in  $(4+3)$ -dimensional unification with  $G=\text{SU}(2)$ , we have  $\hat{R}_G = -3/2$  and  $L_G = 1/\mu$ .

At this point it is worth comparing (6) and (8). Both provide a resolution of the hierarchy problem. But there is a big difference. Clearly (6) does that making the higher-dimensional Newton's constant large, but (8) does that with the dilaton mass. Moreover, the dimension of the internal space  $n$  plays the crucial role in (6). But the curvature of the internal space plays the crucial role in (8). In fact here it is crucial that we have a non-vanishing  $\hat{R}_G$  to resolve the hierarchy problem. Furthermore, (8) solves the hierarchy problem providing a new mass generation mechanism, a geometric mass generation through the curvature of space-time, which tells that the hierarchy problem is closely related to the problem of the origin of mass. More significantly, it tells that the curvature can be the cause (not the effect) of the mass. Understanding the origin of mass has been a fundamental problem in physics. The geometric mass generation mechanism could provide a natural resolution to this problem.

As the scalar graviton the dilaton modifies Einstein's gravitation in a fundamental way [8, 12]. To see how, notice that the sum of the gravitational and fifth force between the two baryonic point particles separated by a distance  $r$  is given in the Newtonian limit as

$$F \simeq \frac{\alpha_g}{r^2} + \frac{\alpha_5}{r^2} e^{-\mu r} = \frac{\alpha_g}{r^2} (1 + \beta e^{-\mu r}), \quad (10)$$

where  $\alpha_g$  and  $\alpha_5$  are the fine structure constants of the gravitation and fifth force, and  $\beta$  is the ratio between them. In terms of Feynman diagrams the first term represents one graviton exchange but the second term represents one dilaton exchange in the zero momentum transfer limit. In Kaluza-Klein unification we have  $\beta = n/(n+2)$  [4, 7], but in general one may assume  $\beta \simeq 1$  because the dilaton is the scalar partner of the graviton. With this assumption one may try to measure the range of the fifth force experimentally. A recent torsion-balance fifth force experiment puts the upper bound of the range to be around  $44 \mu m$  (and the dilaton mass to be around  $4.5 \times 10^{-3} \text{ eV}$ ) with 95% confidence level [14]. This, with (9), implies that in the  $(4+3)$ -dimensional unification with  $G=\text{SU}(2)$  the size of the internal space  $L_G$  can not be larger than  $44 \mu m$ .

On the other hand the cosmology puts a strong theoretical constraint on dilaton mass, because the dilaton can easily be the dominant matter of the universe. In cosmology the dilaton starts with the thermal equilibrium at the beginning and decouples from other sources very early near the Planck time. But it is not stable,

although it will decay very slowly due to the weak (i.e., the gravitational) coupling to the matter fields. There are two dominant decay channels of the dilaton, two-photon decay and fermion-antifermion decay, which may be described by the following Lagrangian [15],

$$\mathcal{L}_{int} \simeq -\frac{1}{4}g_1\sqrt{16\pi G}\hat{\sigma}F_{\mu\nu}F^{\mu\nu} - g_2\sqrt{16\pi G}m\hat{\sigma}\bar{\psi}\psi, \quad (11)$$

where  $g_1$  and  $g_2$  are dimensionless coupling constants,  $m$  is the mass of the fermion, and  $\hat{\sigma} = \sigma/\sqrt{16\pi G}$  is the dimensional (physical) dilaton field. From this we can calculate the two-photon decay rate and the fermion-antifermion decay rate of dilaton in tree approximation [15]

$$\Gamma_{\sigma \rightarrow \gamma\gamma} = \frac{g_1^2 \mu^3}{16 m_p^2},$$

$$\Gamma_{\sigma \rightarrow \bar{\psi}\psi} = \frac{2g_2^2 m^2 \mu}{m_p^2} \times \left[ 1 - \left( \frac{2m}{\mu} \right)^2 \right]^{3/2}. \quad (12)$$

With this we can estimate the present number density of the relic dilaton  $n(t_0)$ ,

$$n(t_0) \simeq 7.5 \exp\left(\frac{-t_0}{\tau(\mu)}\right) \text{ cm}^{-3}, \quad (13)$$

where  $t_0 = 1.5 \times 10^{10}$  yr is the age of the universe and  $\tau(\mu)$  is the total life-time of the dilaton. This implies that the dilaton can easily survive to the present universe and can be the dominant matter of the universe

Now, for the dilaton to be the dark matter, we must have the following constraints for the dilaton mass. First, the dilaton mass density  $\rho(\mu)$  must be equal to the dark matter density  $\rho_0(\mu)$  [15],

$$\rho(\mu) = \mu \times 7.5 \exp\left(\frac{-t_0}{\tau(\mu)}\right) \text{ cm}^{-3}$$

$$\simeq \rho_0(\mu) \simeq 0.23 \times 5.9 \text{ keV cm}^{-3}. \quad (14)$$

Secondly, the energy density of the daughter particles (photons and light fermions)  $\tilde{\rho}(\mu)$  must be negligibly small compared to the dark matter density,

$$\tilde{\rho}(\mu) \ll \rho_0(\mu). \quad (15)$$

To find the dilaton mass which satisfies the above constraints, we have to know the coupling constants  $g_1$  and  $g_2$ . In Kaluza-Klein unification they are given by [4]

$$g_1 = \sqrt{\frac{n+2}{n}}, \quad g_2 = \sqrt{\frac{n}{n+2}}. \quad (16)$$

But here we leave them as free parameters, assuming only  $g_1 \simeq g_2$ .

The first constraint (14) shows that, when  $g_1 \simeq g_2 \simeq 1$ , there are two mass ranges of dilaton in which the relic

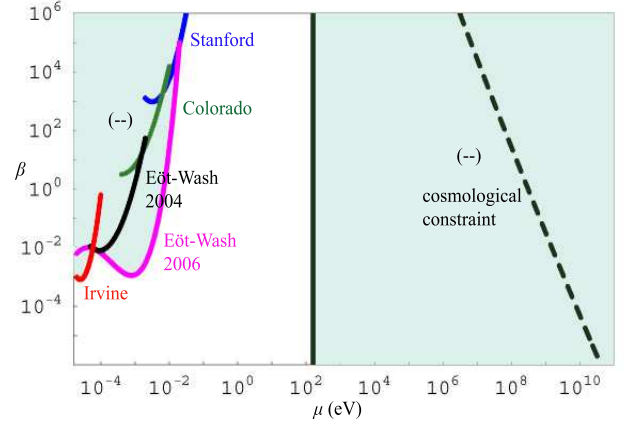


FIG. 1: The allowed mass  $\mu$  of dilaton (uncolored region), where we leave  $\beta = \alpha_5/\alpha_g$  arbitrary. The colored region marked by (--) is the excluded region, and the dotted line represents the mass of the heavy dilaton whose daughter particles overclose the universe.

dilaton could be the dominant matter of the universe;  $\mu \simeq 160 \text{ eV}$  with life-time  $\tau \simeq 3.8 \times 10^{35} \text{ sec}$  and  $\mu \simeq 276 \text{ MeV}$  with life-time  $\tau \simeq 3.3 \times 10^{16} \text{ sec}$ . The dilaton with mass smaller than  $160 \text{ eV}$  survives but fails to be dominant matter due to its low mass, and the dilaton with mass larger than  $276 \text{ MeV}$  does not survive long enough to become the dominant matter of the universe. The dilaton with mass in between is impossible because it would overclose the universe. This clearly rules out the ADD dilaton with mass of  $\text{TeV}$  range.

Moreover, the second constraint (15) shows that the dilaton with mass  $276 \text{ MeV}$  can not be acceptable as the dark matter. To see this notice that  $\tau_2 \simeq 6.9 \times 10^{-2} t_0$ , so that most of the dilaton with mass  $276 \text{ MeV}$  have already decayed. Indeed only  $1.98 \times 10^{-6}$  of the heavy dilatons which survive now fill the dark matter energy. This means that the energy density of the daughter particles from the heavy dilaton must be much bigger than the energy density of the dilaton itself, which tells that the daughter particles from the heavy dilaton overclose the universe. This makes the heavy dilaton unacceptable. On the other hand the dilaton with mass  $160 \text{ eV}$  is almost stable because  $\tau_1 \simeq 8.1 \times 10^{17} t_0$ . In this case the energy density of the daughter particles from the light dilaton is negligible compared to the energy density of the dilaton itself, so that the  $160 \text{ eV}$  dilaton can easily satisfy the second constraint. In conclusion only the  $160 \text{ eV}$  dilaton can become the dark matter of the universe. This puts a strong cosmological constraint on dilaton mass, and immediately rules out the internal space of the scale smaller than  $1.2 \text{ nm}$ .

Putting these constraints together we obtain Fig. 1, which shows the allowed region of the dilaton mass (and the allowed range of the fifth force) in 7-dimensional unification with  $S^3$  compactification of the internal space.

Although  $\beta$  is expected to be of the order one, we leave it arbitrary here. Here we emphasize that the cosmological estimate of the dilaton mass discussed in this paper should be understood as an order estimate, not an exact result, because it is based on the linear approximation. Nevertheless our result demonstrates that cosmology provides a crucial piece of information on the dilaton mass. In particular, cosmology rules out the  $TeV$  scale internal space suggested by ADD. Moreover, our result implies that the dilatonic fifth force may be too short-ranged to be detected by the fifth force experiment. Under this circumstance a totally different type of experiments based on two-photon decay of dilaton is needed to detect the dilaton [17].

In this paper we have discussed how the dilaton mass can resolve the hierarchy problem and explain the origin of mass. Moreover, we have shown how the dilaton can be the dominant matter of the universe, and how the cosmology can determine the dilaton mass the scale of the internal space. In particular, we have shown that the cosmology effectively excludes the dilaton heavier than  $160\text{ eV}$ , and thus the internal space whose scale is smaller than  $10^{-9}\text{ m}$ .

We close with the following remarks.

1. So far we have concentrated on the dilaton. But we emphasize that there are other scalar gravitons called internal gravitons (also called the moduli in string theory) which are described by  $\rho_{ab}$ , which have similar properties [3, 4]. Here we simply mention that in general there are

$(n+2)(n-1)/2$  such internal gravitons which have similar features and thus enhance the impact of the dilaton we have discussed above.

2. In superstring or supergravity the situation is more complicated. For example, in string theory one has to deal with an extra higher-dimensional dilaton (the string dilaton) which remains massless in all orders of perturbation [10]. But once the dilaton acquires a mass, the qualitative features of dilaton physics will remain the same. In particular, the constraint on the dilaton mass shown in Fig.1 must apply to all higher-dimensional unified theories.

3. In the above analysis, we have assumed that  $\langle \sigma \rangle = 0$ . But in cosmology the dilaton field (as the inflaton) may actually evolve in time, so that classically  $\sigma(t_0)$  may not be zero at present time  $t_0$  [7]. Even in this case our conclusion may still be valid, if we identify  $v$  and  $\hat{R}_G$  as the present volume and curvature of the internal space (with  $\sigma(t_0) = 0$ ) [7, 12].

The detailed discussions of our results and related subjects will be published in a separate paper [17].

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